

# Polynomials

## Short Answer Type Questions II [3 Marks]

### Question 1

Find the zeroes of the quadratic polynomial  $3x^2 - 2$  and verify the relationship between the zeroes and the coefficients.

**Answer:**

Given quadratic polynomial is  $3x^2 - 2$

Consider  $p(x) = 3x^2 - 2$

For zeroes of polynomial  $p(x)$ , put  $p(x) = 0$ .

$$\Rightarrow 3x^2 - 2 = 0$$

$$\Rightarrow 3x^2 = 2$$

$$x^2 = \frac{2}{3}$$

$$\Rightarrow x = \pm \sqrt{\frac{2}{3}}$$

Hence zeroes of polynomial  $p(x)$  are  $+\sqrt{\frac{2}{3}}$  and  $-\sqrt{\frac{2}{3}}$

Here  $\alpha = \sqrt{\frac{2}{3}}$  and  $\beta = -\sqrt{\frac{2}{3}}$

$$\text{Hence sum of zeroes} = \alpha + \beta = \sqrt{\frac{2}{3}} - \sqrt{\frac{2}{3}} = 0$$

$$\text{Product of zeroes} = \alpha \beta = \sqrt{\frac{2}{3}} \times \left(-\sqrt{\frac{2}{3}}\right) = -\frac{2}{3}$$

Also from the polynomial  $p(x) = 3x^2 - 2$

$$\text{sum of zeroes} = -\frac{b}{a} = \frac{0}{3} = 0$$

$$\text{Product of zeroes} = \frac{c}{a} = -\frac{2}{3}$$

This verify the relation.

### Question 2

On dividing  $x^3 - 8x^2 + 20x - 10$  by a polynomial  $g(x)$ , the quotient and the remainder were  $x - 4$  and 6 respectively. Find  $g(x)$ .

**Answer:**

Consider Divided:  $f(x) = x^3 - 8x^2 + 20x - 10$

Quotient  $q(x) = x - 4$

remainder  $r(x) = 6$  and divisor is  $g(x)$

Applying division algorithm, we get

$$f(x) = g(x) \cdot q(x) + r(x)$$

$$\Rightarrow g(x) = \frac{f(x) - r(x)}{q(x)} = \frac{(x^3 - 8x^2 + 20x - 10) - (6)}{x - 4}$$

$$= \frac{x^3 - 8x^2 + 20x - 16}{x - 4}$$

So,

$$g(x) = x^2 - 4x + 4$$

$$\begin{array}{r} x^2 - 4x + 4 \\ x - 4 \overline{) x^3 - 8x^2 + 20x - 16} \\ \underline{-x^3 + 4x^2} \phantom{-16} \\ -4x^2 + 20x \phantom{-16} \\ \underline{+4x^2 - 16x} \phantom{-16} \\ 4x - 16 \phantom{-16} \\ \underline{-4x + 16} \\ 0 \end{array}$$

### Question 3

Quadratic polynomial  $2x^2 - 3x + 1$  has zeroes as  $\alpha$  and  $\beta$ . Now form a quadratic polynomial whose zeroes are  $3\alpha$  and  $3\beta$ .

Answer:

$\alpha$  and  $\beta$  are the zeroes of the polynomial  $2x^2 - 3x + 1$

$$\Rightarrow \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{1}{2}$$

Now, zeroes of the required polynomial are  $3\alpha$  and  $3\beta$

$$\Rightarrow S = 3\alpha + 3\beta = 3(\alpha + \beta) = 3\left(\frac{3}{2}\right) = \frac{9}{2}$$

$$\Rightarrow P = (3\alpha)(3\beta) = 9(\alpha\beta) = 9 \times \frac{1}{2} = \frac{9}{2}$$

Now, required polynomial is  $x^2 - Sx + P$

$$= x^2 - \frac{9}{2}x + \frac{9}{2} = \frac{k}{2}(2x^2 - 9x + 9), \text{ where } k \text{ be any constant.}$$

### Question 4.

Divide the polynomial  $x^4 - 11x^2 + 34x - 12$  by  $x - 2$  and find the quotient and the remainder. Also verify the division algorithm.

Answer:

Let

$$p(x) = x^4 - 11x^2 + 34x - 12 \text{ and } g(x) = x - 2$$

$$\begin{array}{r} x^3 + 2x^2 - 13x + 8 \\ x - 2 \overline{) x^4 - 11x^2 + 34x - 12} \\ \underline{-x^4} \phantom{+ 34x - 12} \phantom{+} -2x^3 \\ 2x^3 - 17x^2 \phantom{+ 34x - 12} \\ \underline{-2x^3 + 4x^2} \phantom{+ 34x - 12} \\ -13x^2 + 34x \phantom{- 12} \\ \underline{-13x^2 + 26x} \phantom{- 12} \\ 8x - 12 \\ \underline{-8x + 16} \\ 4 \end{array}$$

Now, quotient

$$q(x) = x^3 + 2x^2 - 13x + 8$$

remainder

$$r(x) = 4$$

By Division algorithm

$$p(x) = g(x) \cdot q(x) + r(x)$$

$$\begin{aligned} x^4 - 11x^2 + 34x - 12 &= (x - 2)(x^3 + 2x^2 - 13x + 8) + 4 \\ &= x^4 + 2x^3 - 13x^2 + 8x - 2x^3 - 4x^2 + 26x - 16 + 4 \\ &= x^4 - 11x^2 + 34x - 12 \end{aligned}$$

Hence division algorithm is verified.

**Question 5.**

An NGO decided to distribute books and pencils to the students of a school running by some other NGO. For this they collected some amount from different people. The total amount collected is represented by  $4x^4 + 2x^3 - 8x^2 + 3x - 7$ . From this fund each student received an equal amount. The number of students, who received the amount, is represented by  $x - 2 + 2x^2$ . After distribution,  $5x - 11$ , amount is left with the NGO which they donated to school for their infrastructure. Find the amount received by each student from the NGO. What value has been depicted here?

**Answer:**

The total amount collected,  $p(x) = 4x^4 + 2x^3 - 8x^2 + 3x - 7$

Number of students,  $g(x) = x - 2 + 2x^2 = 2x^2 + x - 2$

Let amount received by each students,  $q(x)$ .

Amount left after distribution,  $r(x) = 5x - 11$

By using division algorithm, we have

$$p(x) = g(x) \cdot q(x) + r(x)$$

$$4x^4 + 2x^3 - 8x^2 + 3x - 7 = (2x^2 + x - 2) q(x) + (5x - 11)$$

$$\frac{(4x^4 + 2x^3 - 8x^2 + 3x - 7) - (5x - 11)}{2x^2 + x - 2} = q(x)$$

$$q(x) = \frac{4x^4 + 2x^3 - 8x^2 - 2x + 4}{2x^2 + x - 2}$$

$$\begin{array}{r} 2x^2 - 2 \\ 2x^2 + x - 2 \overline{) 4x^4 + 2x^3 - 8x^2 - 2x + 4} \\ \underline{4x^4 + 2x^3 - 4x^2} \phantom{- 2x + 4} \\ -4x^2 - 2x + 4 \\ \underline{-4x^2 - 2x + 4} \\ 0 \end{array}$$

Amount received by each student,  $q(x) = 2x^2 - 2$

**Value:** Humanity and socialism

**Question 6.**

Obtain all other zeroes of the polynomial  $x^4 - 17x^2 - 36x - 20$ , if two of its zeroes are  $+5$  and  $-2$ .

**Answer:**

Consider

$$f(x) = x^4 - 17x^2 - 36x - 20$$

It is given that +5 and -2 are zeroes of polynomial  $f(x)$

$\therefore x = 5$  is zero of polynomial  $f(x)$

$\Rightarrow (x - 5)$  is factor of polynomial  $f(x)$

Similarly  $x = -2$  is a zero of polynomial  $f(x)$ .

$\Rightarrow (x + 2)$  is a factor of polynomial  $f(x)$ .

Hence  $(x - 5)(x + 2)$  is a factor of polynomial  $f(x)$

$\Rightarrow (x^2 - 3x - 10)$  is a factor of polynomial  $f(x)$ .

$\therefore f(x)$  is divisible by  $(x^2 - 3x - 10)$

Now

$$x^4 - 17x^2 - 36x - 20 = (x^2 - 3x - 10)(x^2 + 3x + 2)$$

For other zeroes

Put,

$$x^2 + 3x + 2 = 0$$

$$x^2 + 3x + 2 = x^2 + 2x + x + 2$$

$$= x(x + 2) + 1(x + 2)$$

$$= (x + 2)(x + 1)$$

Put

$$(x + 2)(x + 1) = 0$$

$\Rightarrow$

$$x + 2 = 0 \Rightarrow x = -2$$

or

$$x + 1 = 0 \Rightarrow x = -1$$

$\therefore$  other zeroes are  $x = -2$  and  $x = -1$

$$\begin{array}{r} x^2 + 3x + 2 \\ x^2 - 3x - 10 \overline{) x^4 - 17x^2 - 36x - 20} \\ \underline{-x^4 + 10x^2} \phantom{-} -3x^3 \\ \phantom{-} 3x^3 - 7x^2 - 36x \\ \phantom{-} \underline{-3x^3 + 9x^2} \phantom{-} 30x \\ \phantom{-} \phantom{3x^3 -} 2x^2 - 6x - 20 \\ \phantom{-} \phantom{3x^3 -} \underline{2x^2 - 6x - 20} \\ \phantom{-} \phantom{3x^3 -} \phantom{2x^2 -} 0 \end{array}$$

2015

### Short Answer Type Questions II [3 Marks]

#### Question 7.

Divide the polynomial  $x^4 - 9x^2 + 9$  by the polynomial  $x^2 - 3x$  and verify the division algorithm.

**Answer:**

Here,

$$\text{Dividend} = x^4 - 9x^2 + 9$$

$$\text{Divisor} = x^2 - 3x$$

$$\text{Quotient} = x^2 + 3x$$

$$\text{Remainder} = 9$$

$$\begin{array}{r} x^2 + 3x \\ x^2 - 3x \overline{) x^4 - 9x^2 + 9} \\ \underline{-x^4 + 3x^3} \phantom{+} -3x^3 \\ \phantom{-} 3x^3 - 9x^2 + 9 \\ \phantom{-} \underline{-3x^3 + 9x^2} \phantom{+} 9 \\ \phantom{-} \phantom{3x^3 -} 9 \end{array}$$

#### Verification

By division algorithm we have

$$\text{Dividend} = (\text{Quotient} \times \text{Divisor}) + \text{Remainder}$$

$$= (x^2 + 3x)(x^2 - 3x) + 9$$

$$= x^4 - 9x^2 + 9 = \text{Dividend} = \text{LHS}$$

#### Question 8.

If one zero of the quadratic polynomial  $f(x) = 4x^2 - 8kx + 8x - 9$  is negative of the other, then find the zeroes of  $kx^2 + 3kx + 2$ .

**Answer:**

$$f(x) = 4x^2 - 8kx + 8x - 9 = 4x^2 - (8k - 8)x - 9$$

Let one zero of  $f(x)$  be  $\alpha$  then other zero be  $-\alpha$ .

So, sum of zeroes = 0

$$\Rightarrow \frac{8k - 8}{4} = 0 \Rightarrow 8k - 8 = 0 \Rightarrow k = 1$$

Now, other given polynomial is  $p(x) = kx^2 + 3kx + 2$

$$= x^2 + 3x + 2$$

$$= (x + 2)(x + 1)$$

So, zeroes of  $p(x)$  are -1 and -2.

#### Question 9.

An NGO decided to distribute books and pencils to the students of a school running by some

other NGO. For this, they collected some amount from different number of people. The total amount collected is represented by  $4x^4 + 2x^3 - 8x^2 + 3x - 7$ . The amount is equally divided between each of the students. The number of students, who received the amount is represented by  $x - 2 + 2x^2$ . After distribution,  $5x - 11$ , amount is left with the NGO which they donated to school for their infrastructure. Find the amount received by each student from the NGO.

What value have been depicted here?

**Answer:**

$$\begin{array}{r}
 2x^2 - 2 \\
 2x^2 + x - 2 \overline{) 4x^4 + 2x^3 - 8x^2 + 3x - 7} \\
 \underline{4x^4 + 2x^3 - 4x^2} \phantom{+ 3x - 7} \\
 -4x^2 + 3x - 7 \\
 \underline{-4x^2 - 2x + 4} \phantom{- 7} \\
 + \phantom{- 4x^2} + \phantom{- 2x} - \phantom{4} - 7 \\
 \hline
 5x - 11
 \end{array}$$

So, the amount received by each student = ₹ $(2x^2 - 2)$

The distribution of books and pencils by NGO to school students shows the helping nature of NGO. These activities boost the students.

2014

### Short Answer Type Questions I [2 Marks]

#### Question 10.

If the product of zeroes of the polynomial  $ax^2 - 6x - 6$  is 4, find the value of  $a$ . Find the sum of zeroes of the polynomial.

**Answer:**

Let  $\alpha, \beta$  be the zeroes of given polynomial  $p(x) = ax^2 - 6x - 6$ .

Then,

$$\alpha\beta = \frac{-6}{a} \Rightarrow 4 = \frac{-6}{a} \Rightarrow a = \frac{-6}{4} = \frac{-3}{2}$$

Thus,

$$a = \frac{-3}{2}$$

$$\text{Now, sum of zeroes} = \frac{6}{a} = \frac{6}{\left(\frac{-3}{2}\right)} = \frac{2 \times 6}{-3} = -4$$

#### Question 11.

Find the zeroes of the quadratic polynomial  $9t^2 - 6t + 1$  and verify the relationship between the zeroes and the coefficients.

**Answer:**

$$f(t) = 9t^2 - 6t + 1 = (3t - 1)^2$$

So,  $\frac{1}{3}, \frac{1}{3}$  are the zeroes of  $f(t)$ .

$$\text{Now, sum of zeroes} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} = -\frac{(\text{coefficient of } t)}{(\text{coefficient of } t^2)}$$

$$\text{and product of zeroes} = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{9} = \frac{\text{constant term}}{\text{coefficient of } t^2}$$

Hence, verified the relationship between the zeroes and the coefficients.

#### Question 12.

When a polynomial  $6x^4 + 8x^3 + 290x^2 + 21x + 7$  is divided by another polynomial  $3x^2 + 4x + 1$  the remainder is in the form  $ax + b$ . Find  $a$  and  $b$ .

Answer:

$$\begin{array}{r}
 2x^2 + 9 \\
 3x^2 + 4x + 1 \overline{) 6x^4 + 8x^3 + 29x^2 + 21x + 7} \\
 \underline{6x^4 + 8x^3 + 2x^2} \phantom{+ 7} \\
 27x^2 + 21x + 7 \\
 \underline{27x^2 + 36x + 9} \\
 -15x - 2
 \end{array}$$

$\therefore ax + b$  is given as remainder.

$$\text{So, } ax + b = -15x - 2 \Rightarrow a = -15 \text{ and } b = -2$$

### Short Answer Type Questions II [3 Marks]

#### Question 13.

Obtain all other zeroes of the polynomial  $x^4 + 4x^3 - 2x^2 - 20x - 15$  if two of its zeroes are  $\sqrt{5}$  and  $-\sqrt{5}$

Answer:

$\therefore \sqrt{5}$  and  $-\sqrt{5}$  are the zeroes of  $p(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$ .

$\therefore (x - \sqrt{5})(x + \sqrt{5})$  i.e.  $(x^2 - 5)$  is the factor of  $p(x)$ .

To find other factor,

$$\begin{array}{r}
 x^2 + 4x + 3 \\
 x^2 - 5 \overline{) x^4 + 4x^3 - 2x^2 - 20x - 15} \\
 \underline{x^4 \phantom{+ 4x^3} - 5x^2} \phantom{- 20x - 15} \\
 4x^3 + 3x^2 - 20x - 15 \\
 \underline{4x^3 \phantom{+ 3x^2} - 20x} \phantom{- 15} \\
 3x^2 - 15 \\
 \underline{3x^2 \phantom{- 15}} \\
 0
 \end{array}$$

$$\begin{aligned}
 \text{So, } x^4 + 4x^3 - 2x^2 - 20x - 15 &= (x^2 - 5)(x^2 + 4x + 3) \\
 &= (x^2 - 5)(x + 3)(x + 1)
 \end{aligned}$$

$\therefore$  Other zeroes of  $p(x)$  are  $-1$  and  $-3$ .

#### Question 14.

If  $\alpha$  and  $\beta$  are zeroes of a polynomial  $x^2 + 6x + 9$ , then form a polynomial whose zeroes are  $-\alpha$  and  $-\beta$ .

Answer:

$$\text{Let } p(x) = x^2 + 6x + 9$$

$\therefore \alpha, \beta$  are the zeroes of  $p(x)$

So, sum of zeroes,  $\alpha + \beta = -6$  and product of zeroes  $\alpha\beta = 9$ .

Now,  $-\alpha$  and  $-\beta$  are the zeroes of required quadratic polynomial.

So, sum of zeroes of required polynomial  $= -\alpha - \beta = -(\alpha + \beta) = +6$

and product of zeroes of required polynomial  $= (-\alpha)(-\beta) = \alpha\beta = 9$ .

$\therefore$  Required quadratic polynomial is given by

$$\begin{aligned}
 q(x) &= x^2 - (\text{sum of zeroes})x + \text{product of zeroes} \\
 &= x^2 - 6x + 9
 \end{aligned}$$

#### Question 15.

Find the zeroes of the quadratic polynomial  $3x^2 - 2$  and verify the relationship between the

zeroes and the coefficients.

**Answer:**

$$p(x) = 3x^2 - 2 = (\sqrt{3}x)^2 - (\sqrt{2})^2 = (\sqrt{3}x - \sqrt{2})(\sqrt{3}x + \sqrt{2})$$

So, zeroes of  $p(x)$  are  $\frac{\sqrt{2}}{\sqrt{3}}$  and  $-\frac{\sqrt{2}}{\sqrt{3}}$ .

Now,

$$\text{Sum of zeroes} = \frac{\sqrt{2}}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}} = 0 = -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{and product of zeroes} = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)\left(-\frac{\sqrt{2}}{\sqrt{3}}\right) = \frac{-2}{3} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

Hence, verified the relationship between the zeroes and the coefficients.

#### Question 16.

If a polynomial  $x^4 - 3x^3 - 6x^2 + kx - 16$  is exactly divisible by  $x^2 - 3x + 2$ , then find the value of  $k$ .

**Answer:**

$$\text{Let } p(x) = x^4 - 3x^3 - 6x^2 + kx - 16 \text{ and } g(x) = x^2 - 3x + 2 = (x - 1)(x - 2)$$

$\therefore p(x)$  is divisible by  $g(x)$

So, 1 and 2 are zeroes of  $p(x)$ .

$$\therefore p(1) = 0$$

$$\Rightarrow (1)^4 - 3(1)^3 - 6(1)^2 + k(1) - 16 = 0$$

$$\Rightarrow 1 - 3 - 6 + k - 16 = 0$$

$$\Rightarrow k - 24 = 0$$

$$\Rightarrow k = 24$$

Thus, the value of  $k$  is 24.

#### Question 17.

Obtain all other zeroes of the polynomial  $x^4 - 17x^2 - 36x - 20$ , if two of its zeroes are 5 and -2.

**Answer:**

$$\therefore 5 \text{ and } -2 \text{ are the zeroes of } p(x) = x^4 - 17x^2 - 36x - 20$$

$$\therefore (x - 5)(x + 2) \text{ or } (x^2 - 3x - 10) \text{ is the factor of } p(x).$$

$$\begin{array}{r} x^2 + 3x + 2 \\ x^2 - 3x - 10 \overline{) x^4 - 17x^2 - 36x - 20} \\ \underline{x^4 - 10x^2 \phantom{- 36x} - 3x^3} \phantom{- 20} \\ - \phantom{x^4} + \phantom{x^3} + \phantom{x^2} \phantom{- 36x} \phantom{- 20} \\ \underline{3x^3 - 7x^2 - 36x - 20} \\ 3x^3 - 9x^2 - 30x \phantom{- 20} \\ \underline{- \phantom{x^3} + \phantom{x^2} +} \phantom{- 36x} \phantom{- 20} \\ 2x^2 - 6x - 20 \\ \underline{2x^2 - 6x - 20} \\ - \phantom{x^2} + \phantom{x} + \phantom{- 20} \\ \underline{\phantom{x^2} \phantom{x} \phantom{- 20}} \\ 0 \end{array}$$

$$\begin{aligned} \text{So, } x^4 - 17x^2 - 36x - 20 &= (x^2 - 3x - 10)(x^2 + 3x + 2) \\ &= (x^2 - 3x - 10)(x + 1)(x + 2) \end{aligned}$$

$\therefore$  Other zeroes of  $p(x)$  are -2 and -1.

#### Question 18.

Obtain all other zeroes of the polynomial  $x^4 - 3\sqrt{2}x^3 - 3x^2 + 3\sqrt{2}x - 4$ , if two of its zeroes are  $\sqrt{2}$  and  $2\sqrt{2}$ .

**Answer:**

$\therefore \sqrt{2}$  and  $2\sqrt{2}$  are the zeroes of  $p(x) = x^4 - 3\sqrt{2}x^3 + 3x^2 + 3\sqrt{2}x - 4$

$\therefore (x - \sqrt{2})(x - 2\sqrt{2})$  or  $(x^2 - 3\sqrt{2}x + 4)$  is the factor of  $p(x)$

$$\begin{array}{r}
 x^2 - 1 \\
 x^2 - 3\sqrt{2}x + 4 \overline{) x^4 - 3\sqrt{2}x^3 + 3x^2 + 3\sqrt{2}x - 4} \\
 \underline{x^4 - 3\sqrt{2}x^3 + 4x^2} \phantom{- 4} \\
 -x^2 + 3\sqrt{2}x - 4 \\
 \underline{-x^2 + 3\sqrt{2}x - 4} \\
 0
 \end{array}$$

So,  $x^4 - 3\sqrt{2}x^3 + 3x^2 + 3\sqrt{2}x - 4 = (x^2 - 3\sqrt{2}x + 4)(x^2 - 1) = (x^2 - 3\sqrt{2}x + 4)(x - 1)(x + 1)$

$\therefore$  Other zeroes of  $p(x)$  are  $-1$  and  $1$ .

### Question 19.

Divide the polynomial  $3x^3 - 2x^2 + 5x - 5$  by  $3x + 1$  and verify the division algorithm.

**Answer:**

Here, Dividend =  $3x^3 - 2x^2 + 5x - 5$

Divisor =  $3x + 1$

Quotient =  $x^2 - x + 2$

Remainder =  $-7$

Division algorithm is

Dividend = (Divisor)  $\times$  (Quotient) + Remainder

Now, RHS = (Divisor)  $\times$  (Quotient) + (Remainder)

$$= (3x + 1)(x^2 - x + 2) + (-7)$$

$$= 3x^3 - 3x^2 + 6x + x^2 - x + 2 - 7$$

$$= 3x^3 - 2x^2 + 5x - 5 = \text{Dividend} = \text{LHS}$$

Hence, division algorithm is verified.

$$\begin{array}{r}
 x^2 - x + 2 \\
 3x + 1 \overline{) 3x^3 - 2x^2 + 5x - 5} \\
 \underline{3x^3 + x^2} \phantom{- 5} \\
 -3x^2 + 5x - 5 \\
 \underline{-3x^2 - x} \phantom{- 5} \\
 6x - 5 \\
 \underline{6x + 2} \\
 -7
 \end{array}$$

2013

### Short Answer Type Questions I [2 Marks]

### Question 20.

Find a quadratic polynomial whose zeroes are  $3 + \sqrt{2}$  and  $3 - \sqrt{2}$ .

**Answer:**

$$\text{Sum of zeroes} = (3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$$

$$\text{Product of zeroes} = (3 + \sqrt{2})(3 - \sqrt{2}) = 7$$

$\therefore$  Required quadratic polynomial is  $x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$ , i.e.  $x^2 - 6x + 7$ .

### Question 21.

$$\frac{3 + \sqrt{5}}{5} \text{ and } \frac{3 - \sqrt{5}}{5}$$

Find a quadratic polynomial whose zeroes are

**Answer:**

$$\text{Sum of zeroes} = \frac{3 + \sqrt{5}}{5} + \frac{3 - \sqrt{5}}{5} = \frac{6}{5}$$

$$\text{Product of zeroes} = \left(\frac{3 + \sqrt{5}}{5}\right)\left(\frac{3 - \sqrt{5}}{5}\right) = \frac{9 - 5}{25} = \frac{4}{25}$$

$\therefore$  Required quadratic polynomial is  $x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$ ,

$$\text{i.e. } x^2 - \frac{6}{5}x + \frac{4}{25} \text{ or } 25x^2 - 6x + 4.$$

### Question 22.

Verify whether  $2, 3$  and  $1/2$  are the zeroes of the polynomial



**Answer:**

First get the factors of  $p(x)$ .

$$p(x) = 2x^3 - 11x^2 + 17x - 6 = (x-2)(x-3)(2x-1)$$

So, zeroes of  $p(x)$  are 2, 3 and  $\frac{1}{2}$ .

**Alternative Method:**

$$p(x) = 2x^3 - 11x^2 + 17x - 6$$

Now,

$$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6 = 16 - 44 + 34 - 6 = 0$$

$$p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6 = 54 - 99 + 51 - 6 = 0$$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6 = \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6 = 0$$

$\therefore p(2) = p(3) = p\left(\frac{1}{2}\right) = 0 \therefore 2, 3 \text{ and } \frac{1}{2} \text{ are the zeroes of } p(x).$

### Question 23.

Obtain all other zeroes of the polynomial  $x^4 + 4x^3 - 2x^2 - 20x - 15$  if two of its zeroes are  $\sqrt{5}$  and  $-\sqrt{5}$ .

**Answer:**

Here,

$$\text{Dividend} = x^4 - 9x^2 + 9$$

$$\text{Divisor} = x^2 - 3x$$

$$\text{Quotient} = x^2 + 3x$$

$$\text{Remainder} = 9$$

$$\begin{array}{r} x^2 + 3x \\ x^2 - 3 \overline{) x^4 - 9x^2 + 9} \\ \underline{x^4 \phantom{- 9x^2} - 3x^3} \phantom{+ 9} \\ - \phantom{3x^3} - 9x^2 + 9 \\ \underline{3x^3 - 9x^2} \phantom{+ 9} \\ - \phantom{3x^3} + \phantom{9x^2} 9 \\ \underline{\phantom{3x^3} + \phantom{9x^2} 9} \\ 9 \end{array}$$

**Verification**

By division algorithm we have

$$\text{Dividend} = (\text{Quotient} \times \text{Divisor}) + \text{Remainder}$$

$$= (x^2 + 3x)(x^2 - 3x) + 9$$

$$= x^4 - 9x^2 + 9 = \text{Dividend} = \text{LHS}$$

2012

### Short Answer Type Questions I [2 Marks]

### Question 24.

If the zeroes of the polynomial  $x^2 + px + q$  are double in value to the zeroes of  $2x^2 - 5x - 3$ , find the value of  $p$  and  $q$ .

**Answer:**

Let  $\alpha, \beta$  are zeroes of  $2x^2 - 5x - 3$ .

$$\therefore \alpha + \beta = \frac{5}{2}, \alpha\beta = \frac{-3}{2}$$

A.T.Q.,

It is given that  $2\alpha$  and  $2\beta$  are zeroes of  $x^2 + px + q$

$$\therefore 2\alpha + 2\beta = -p \Rightarrow 2 \times (\alpha + \beta) = -p$$

$$2 \times \frac{5}{2} = -p \Rightarrow p = -5$$

$$\text{and } 2\alpha \times 2\beta = q \Rightarrow 4\alpha\beta = q$$

$$\Rightarrow 4 \times \left(\frac{-3}{2}\right) = q \Rightarrow q = -6$$

### Question 25.

Show that  $\frac{1}{2}$  and  $-\frac{3}{2}$  are the zeroes of the polynomial  $4x^2 + 4x - 3$  and verify the relationship between zeroes and coefficients of polynomial.

**Answer:**



Here  $p(x) = 4x^2 + 4x - 3 = (2x + 3)(2x - 1)$

∴ Zeroes of  $p(x)$  are  $\frac{1}{2}$  and  $\frac{-3}{2}$ .

Now,

$$\text{Sum of zeroes} = \frac{1}{2} - \frac{3}{2} = -1 = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{and product of zeroes} = \left(\frac{1}{2}\right)\left(\frac{-3}{2}\right) = \frac{-3}{4} = \frac{(\text{constant term})}{(\text{coefficient of } x^2)}$$

Hence, verified the relationships between the zeroes and the coefficients.

#### Question 26.

Find the value of  $b$  for which  $(2x + 3)$  is a factor of  $2x^3 + 9x^2 - x - b$ .

**Answer:**

$$p(x) = 2x^3 + 9x^2 - x - b$$

$2x + 3$  is a factor of  $p(x)$ .

$$\therefore p\left(\frac{-3}{2}\right) = 0 \Rightarrow 2\left(\frac{-3}{2}\right)^3 + 9\left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right) - b = 0$$

$$\Rightarrow 2 \times \left(\frac{-27}{8}\right) + \frac{81}{4} + \frac{3}{2} - b = 0 \Rightarrow b = \frac{-27}{4} + \frac{81}{4} + \frac{3}{2} = 15$$

#### Question 27.

Given that  $x - \sqrt{5}$  is factor of the polynomial  $x^3 - 3 - \sqrt{5}x^2 - 5x + 15\sqrt{5}$ , find, all the zeroes of the polynomial.

**Answer:**

$$\begin{array}{r} x^2 - 2\sqrt{5}x - 15 \\ x - \sqrt{5} \overline{) x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5}} \\ \underline{x^3 - \sqrt{5}x^2} \phantom{- 5x + 15\sqrt{5}} \\ -2\sqrt{5}x^2 - 5x + 15\sqrt{5} \\ \underline{-2\sqrt{5}x^2 + 10x} \phantom{+ 15\sqrt{5}} \\ -15x + 15\sqrt{5} \\ \underline{-15x + 15\sqrt{5}} \\ 0 \end{array}$$

#### Question 28.

If the polynomial  $x^4 - 6x^3 + 16x^2 - 25x + 10$  is divided by  $(x^2 - 2x + k)$  the remainder comes out to be  $x + a$ , find  $k$  and  $a$ .

**Answer:**

$$P(x) = x^4 - 6x^3 + 16x^2 - 25x + 10,$$

$$g(x) = x^2 - 2x + k$$

$$\begin{array}{r}
 x^2 - 4x + (8 - k) \\
 x^2 - 2x + k \overline{) x^4 - 6x^3 + 16x^2 - 25x + 10} \\
 \underline{x^4 - 2x^3 + kx^2} \phantom{- 25x + 10} \\
 -4x^3 + (16 - k)x^2 - 25x + 10 \\
 \underline{-4x^3 + 8x^2} \phantom{- 25x + 10} \\
 (8 - k)x^2 + (4k - 25)x + 10 \\
 \underline{(8 - k)x^2 - (16 - 2k)x + 8k - k^2} \\
 (2k - 9)x + (10 - 8k - k^2)
 \end{array}$$

∴ Remainder is given as  $x + a$

$$\therefore x + a = (2k - 9)x + (10 - 8k + k^2)$$

$$\Rightarrow 2k - 9 = 1$$

$$\text{and } k^2 - 8k + 10 = a$$

$$\Rightarrow 2k = 10$$

$$\Rightarrow k = 5$$

$$\text{and } 5^2 - 8(5) + 10 = a$$

$$\Rightarrow a = 25 - 40 + 10$$

$$\Rightarrow a = -5$$

Thus,  $a = -5$  and  $k = 5$

#### Question 29.

What must be subtracted or added to  $p(x) = 8x^4 + 14x^3 - 2x^2 + 8x - 12$  so that  $4x^2 + 3x - 2$  is a factor of  $p(x)$ ?

**Answer:**

$$\begin{array}{r}
 2x^2 + 2x - 1 \\
 4x^2 + 3x - 2 \overline{) 8x^4 + 14x^3 - 2x^2 + 8x - 12} \\
 \underline{8x^4 + 6x^3 - 4x^2} \phantom{+ 8x - 12} \\
 8x^3 + 2x^2 + 8x - 12 \\
 \underline{8x^3 + 6x^2 - 4x} \phantom{- 12} \\
 -4x^2 + 12x - 12 \\
 \underline{-4x^2 - 3x + 2} \\
 15x - 14
 \end{array}$$

Remainder =  $15x - 14$

∴ If we subtract  $15x - 14$  or add  $-15x + 14$  then remainder will be 0.

Then  $4x^2 + 3x - 2$  will be a factor of given polynomial.

2011

#### Short Answer Type Questions I [2 Marks]

#### Question 30.

Divide  $x^4 - 3x^2 + 4x + 5$  by  $x^2 - x + 1$ , find quotient and remainder.

**Answer:**

$$\begin{array}{r}
 x^2 + x - 3 \\
 x^2 - x + 1 \overline{) x^4 - 3x^2 + 4x + 5} \\
 \underline{x^4 + x^2 \phantom{+ 4x + 5} - x^3} \\
 - \phantom{x^4} - \phantom{x^2} + \phantom{4x + 5} \\
 x^3 - 4x^2 + 4x + 5 \\
 \underline{x^3 - x^2 + x} \\
 - \phantom{x^3} + \phantom{x^2} - \phantom{4x + 5} \\
 - 3x^2 + 3x + 5 \\
 \underline{- 3x^2 + 3x - 3} \\
 + \phantom{- 3x^2} - \phantom{3x} + \phantom{5} \\
 8
 \end{array}$$

Quotient =  $x^2 + x - 3$ , Remainder = 8

### Question 31.

If 2 and -3 are the zeroes of the quadratic polynomial  $x^2 + (a + 1)x + b$ ; then find the values of a and b.

**Answer:**

$$p(x) = x^2 + (a + 1)x + b$$

$\therefore$  2 is a zero of  $p(x)$

$$\Rightarrow p(2) = 0$$

$$\Rightarrow 2^2 + (a + 1)2 + b = 0$$

$$\Rightarrow 2a + b = -6$$

$$\text{Also, } p(-3) = 0$$

$$\Rightarrow (-3)^2 + (a + 1)(-3) + b = 0$$

$$\Rightarrow -3a + b = -6$$

Solving equation (i) and (ii), we get  $a = 0, b = -6$

### Question 32.

It being given that 1 is one of the zeroes of the polynomial  $7x - x^3 - 6$ . Find its other zeroes.

**Answer:**

$$\begin{array}{r}
 -x^2 - x + 6 \\
 x - 1 \overline{) -x^3 + 7x - 6} \\
 \underline{-x^3 \phantom{+ 7x - 6} + x^2} \\
 + \phantom{-x^3} - \phantom{7x - 6} \\
 -x^2 + 7x - 6 \\
 \underline{-x^2 + x} \\
 + \phantom{-x^2} - \phantom{7x - 6} \\
 6x - 6 \\
 \underline{6x - 6} \\
 - \phantom{6x - 6} + \phantom{6} \\
 0
 \end{array}$$

$$p(x) = 7x - x^3 - 6$$

$\therefore$  1 is a zero of  $p(x) \Rightarrow (x - 1)$  is a factor of  $p(x)$

For other zeroes,

$$\begin{aligned}
 \therefore 7x - x^3 - 6 &= (x - 1)(-x^2 - x + 6) \\
 &= (1 - x)(x^2 + x - 6) \\
 &= (1 - x)(x + 3)(x - 2)
 \end{aligned}$$

$\therefore$  Other two zeroes of  $p(x)$  are 2 and -3.

### Short Answer Type Questions II [3 Marks]

### Question 33.

If the polynomial  $6x^4 + 8x^3 + 17x^2 + 21x + 7$  is divided by another polynomial  $3x^2 + 4x + 1$ , then what will be the quotient and remainder?

**Answer:**

$$\begin{array}{r}
 2x^2 + 5 \\
 3x^2 + 4x + 1 \overline{) 6x^4 + 8x^3 + 17x^2 + 21x + 7} \\
 \underline{6x^4 + 8x^3 + 2x^2} \phantom{+ 7} \\
 15x^2 + 21x + 7 \\
 \underline{15x^2 + 20x + 5} \\
 x + 2
 \end{array}$$

Quotient =  $2x^2 + 5$ , remainder =  $x + 2$

#### Question 34

On dividing the polynomial  $4x^4 - 5x^3 - 39x^2 - 46x - 2$  by the polynomial  $g(x)$ , the quotient and remainder were  $x^2 - 3x - 5$  and  $-5x + 8$  respectively. Find  $g(x)$ .

**Answer:**

$$p(x) = 4x^4 - 5x^3 - 39x^2 - 46x - 2$$

$$q(x) = x^2 - 3x - 5, r(x) = -5x + 8$$

According to division algorithm,

$$p(x) = g(x) \cdot q(x) + r(x) \Rightarrow p(x) - r(x) = g(x) \cdot q(x)$$

$$\Rightarrow \frac{p(x) - r(x)}{q(x)} = g(x)$$

$$\begin{aligned}
 \Rightarrow g(x) &= \frac{(4x^4 - 5x^3 - 39x^2 - 46x - 2) - (-5x + 8)}{x^2 - 3x - 5} = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{x^2 - 3x - 5} \\
 &= 4x^2 + 7x + 2
 \end{aligned}$$

#### Long Answer Type Questions [4 Marks]

#### Question 35.

Find other zeroes of the polynomial  $x^4 - 7x^2 + 12$  if it is given that two of its zeroes are  $\sqrt{3}$  and  $-\sqrt{3}$ .

**Answer:**

$$p(x) = x^4 - 7x^2 + 12$$

$\therefore \sqrt{3}$  and  $-\sqrt{3}$  are the zeroes of  $p(x)$ ,

$\therefore (x - \sqrt{3})(x + \sqrt{3})$  is a factor of  $p(x)$

$\Rightarrow x^2 - 3$  is a factor of  $p(x)$ .

For other factors of  $p(x)$ ,

$$\therefore x^4 - 7x^2 + 12 = (x^2 - 3)(x^2 - 4) = (x^2 - 3)(x - 2)(x + 2)$$

$\therefore$  Other two zeroes are 2 and -2.

$$\begin{array}{r}
 x^2 - 4 \\
 x^2 - 3 \overline{) x^4 - 7x^2 + 12} \\
 \underline{x^4 - 3x^2} \phantom{+ 12} \\
 -4x^2 + 12 \\
 \underline{-4x^2 + 12} \\
 0
 \end{array}$$

#### Question 36.

Obtain all other zeroes of the polynomial  $x^4 - 3x^3 - x^2 + 9x - 6$ , if two of its zeroes are  $\sqrt{3}$  and  $-\sqrt{3}$ .

**Answer:**

$$p(x) = x^4 - 3x^3 - x^2 + 9x - 6$$

$\therefore \sqrt{3}$  and  $-\sqrt{3}$  are the zeroes of  $p(x)$

$\therefore (x - \sqrt{3})(x + \sqrt{3})$  is a factor of  $p(x)$

$\Rightarrow x^2 - 3$  is a factor of  $p(x)$

For other factor

$$\begin{aligned}
 \therefore p(x) &= (x^2 - 3)(x^2 - 3x + 2) \\
 &= (x^2 - 3)(x - 2)(x - 1)
 \end{aligned}$$

$\therefore$  Other two zeroes are 1 and 2.

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 x^2 - 3 \overline{) x^4 - 3x^3 - x^2 + 9x - 6} \\
 \underline{x^4 - 3x^3} \phantom{- x^2 + 9x - 6} \\
 -x^2 + 9x - 6 \\
 \underline{-x^2 + 3x - 2} \\
 6x - 4 \\
 \underline{6x - 4} \\
 0
 \end{array}$$

#### Question 37.

Divide  $2x^4 - 9x^3 + 5x^2 + 3x - 8$  by  $x^2 - 4x + 1$  and verify the division algorithm.

## Checking

$$g(x) = x^2 - 4x + 1$$

$$= 2x^4 - 9x^3 + 5x^2 + 3x - 8 = p(x)$$

$$\begin{array}{r}
 2x^2 - x - 1 \\
 x^2 - 4x + 1 \overline{) 2x^4 - 9x^3 + 5x^2 + 3x - 8} \\
 \underline{2x^4 - 8x^3 + 2x^2} \phantom{+ 3x - 8} \\
 -x^3 + 3x^2 + 3x - 8 \\
 \underline{-x^3 + 4x^2 - x} \phantom{- 8} \\
 + \phantom{-} - \phantom{+} + \\
 \phantom{+} -x^2 + 4x - 8 \\
 \phantom{+} -x^2 + 4x - 1 \\
 \phantom{+} \phantom{-} + \phantom{-} + \\
 \phantom{+} \phantom{-} \phantom{+} -7
 \end{array}$$

### Very Short Answer Type Questions [1 Mark]

**Answer:**

$$g(x) = k(x^2 - 6x + 4)$$

**Answer:**

$$\therefore \alpha + \beta = \frac{-7}{2}$$

$$\alpha\beta = \frac{5}{2}$$

$$\therefore \alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = \frac{-2}{2} = -1.$$

**Answer:**

$$\therefore (2 + \sqrt{3}) + \alpha = -\left(\frac{-4}{1}\right)$$

$$\Rightarrow \alpha = 4 - 2 - \sqrt{3} = 2 - \sqrt{3}.$$

**Answer:**



**Question 46.**

For what value of  $p$ ,  $(-4)$  is a zero of the polynomial  $x^2 - 2x - (7p + 3)$

**Answer:**

$$\begin{aligned} \because (-4) \text{ is a zero of polynomial } p(x) &= x^2 - 2x - (7p + 3) \\ \therefore p(-4) &= 0 \\ \text{Hence, } (-4)^2 - 2(-4) - (7p + 3) &= 0 \\ 16 + 8 - 7p - 3 &= 0 \\ 21 - 7p &= 0 \Rightarrow p = \frac{21}{7} = 3 \end{aligned}$$

**Question 47.**

If 1 is a zero of polynomial  $p(x) = ax^2 - 3(a - 1) - 1$ , then find the value of  $a$ .

**Answer:**

$$\begin{aligned} \because 1 \text{ is a zero of } p(x) \\ \Rightarrow p(1) &= 0 \Rightarrow a(1)^2 - 3(a - 1) - 1 = 0 \\ \Rightarrow a - 3a + 3 - 1 &= 0 \Rightarrow a = 1 \end{aligned}$$

**Question 48.**

Write the polynomial, the product and sum of whose zeroes are  $-9/2$  and  $-3/2$  respectively.

**Answer:**

Required polynomial is given by

$$p(x) = x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$$

$$\text{Here, sum of zeroes} = \frac{-3}{2} \text{ and product of zeroes} = \frac{-9}{2}$$

$$\therefore p(x) = x^2 - \left(\frac{-3}{2}\right)x + \left(\frac{-9}{2}\right) = x^2 + \frac{3}{2}x - \frac{9}{2}$$

**Short Answer type Questions I [2 marks]****Question 49.**

If the polynomial  $6x^4 + 8x^3 + 17x^2 + 21x + 7$  is divided by another polynomial  $3x^2 + 4x + 1$ , the remainder comes out to be  $(ax + b)$ , find  $a$  and  $b$ .

**Answer:**

The given remainder is  $ax + b$ .

$$\text{then } ax + b = x + 2$$

$$\text{So, } a = 1, b = 2.$$

$$\begin{array}{r} 2x^2 + 5 \\ 3x^2 + 4x + 1 \overline{) 6x^4 + 8x^3 + 17x^2 + 21x + 7} \\ \underline{6x^4 + 8x^3 + 2x^2} \phantom{+ 7} \\ 15x^2 + 21x + 7 \\ \underline{15x^2 + 20x + 5} \\ x + 2 \end{array}$$

**Question 50.**

If the polynomial  $x^4 + 2x^3 + 8x^2 + 12x + 18$  is divided by another polynomial  $x^2 + 5$ , the remainder comes out to be  $px + q$ , find values of  $p$  and  $q$ .

**Answer:**

$$\text{Polynomial } p(x) = x^4 + 2x^3 + 8x^2 + 12x + 18$$

is divided by  $x^2 + 5$ .

Remainder is  $2x + 3$

$$\therefore 2x + 3 = px + q$$

$$\text{Hence, } p = 2 \text{ and } q = 3$$

$$\begin{array}{r} x^2 + 2x + 3 \\ x^2 + 5 \overline{) x^4 + 2x^3 + 8x^2 + 12x + 18} \\ \underline{x^4 \phantom{+ 2x^3} + 5x^2} \phantom{+ 12x + 18} \\ 2x^3 + 3x^2 + 12x + 18 \\ \underline{2x^3 \phantom{+ 3x^2} + 10x} \phantom{+ 18} \\ 3x^2 + 2x + 18 \\ \underline{3x^2 \phantom{+ 2x} + 15} \\ 2x + 3 \end{array}$$



**Question 51.**

Find all the zeroes of the polynomial  $x^3 + 3x^2 - 2x - 6$ , if two of its zeroes are  $-\sqrt{2}$  and  $\sqrt{2}$ .

**Answer:**

$$p(x) = x^3 + 3x^2 - 2x - 6$$

$\therefore -\sqrt{2}$  and  $\sqrt{2}$  are zeroes of  $p(x)$ .

$\therefore \{x - (-\sqrt{2})\} (x - \sqrt{2})$  is a factor of  $p(x)$ .

$\Rightarrow x^2 - 2$  is a factor of  $p(x)$ .

$$\therefore x^3 + 3x^2 - 2x - 6 = (x^2 - 2)(x + 3)$$

$\therefore$  Other zero is  $-3$ ,

$\therefore$  All the zeroes of  $p(x)$  are  $-\sqrt{2}$ ,  $\sqrt{2}$  and  $-3$ .

$$\begin{array}{r} x+3 \\ x^2-2 \overline{) x^3+3x^2-2x-6} \\ \underline{-x^3} \phantom{+} -2x \\ \phantom{-} 3x^2 \phantom{-} -6 \\ \phantom{-} 3x^2 \phantom{-} -6 \\ \underline{\phantom{-}} \phantom{+} 0 \end{array}$$

**Question 52.**

Find all the zeroes of the polynomial  $2x^3 + x^2 - 6x - 3$ , if two of its zeroes are  $-\sqrt{3}$  and  $\sqrt{3}$ .

**Answer:**

$$\text{Let } p(x) = 2x^3 + x^2 - 6x - 3$$

$$= x^2(2x + 1) - 3(2x + 1) = (x^2 - 3)(2x + 1) = (x + \sqrt{3})(x - \sqrt{3})(2x + 1)$$

$-\sqrt{3}$  and  $\sqrt{3}$  are two zeroes of  $p(x)$  (given)

$\therefore$  All the zeroes of given polynomial are  $-\sqrt{3}$ ,  $\sqrt{3}$  and  $-\frac{1}{2}$ .